## Game Theory Discussion Week 5

## Problem 1: Evolutionary Games

Consider the following payoff table.

|  | Tortoise | Hare |
| :---: | :---: | :---: |
| Tortoise | $\mathrm{c}, \mathrm{c}$ | $-2,1$ |
| Hare | $1,-2$ | 0,0 |

(a) For what values of $c$ are tortoises fitter than hares if $t$, the proportion of tortoises in the population, is 0.5 ?
(b) For what values of $c$ are tortoises fitter than hares if $t=0.1$ ?
(c) If $c=1$, will a single hare successfully invade a population of pure tortoises? Explain why or why not.
(d) In terms of $t$, how large must $c$ be for tortoises to be fitter than hares?
(e) In terms of $c$, what is the level of $t$ in a polymorphic equilibrium? For what values of $c$ will such an equilibrium exist?

## Problem 2: Strategy and Voting

Consider the following table, which gives the ballots of a small town of seven citizens voting on five policy proposals put forward by the mayor.

|  | Anderson | Brown | Clark | Davis | Evans | Foster | Garcia |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | V | V | W | W | X | Y | Z |
| 2nd | W | X | V | X | Y | X | Y |
| 3rd | X | W | Y | V | Z | Z | X |
| 4th | Y | Y | X | Y | V | W | W |
| 5th | Z | Z | Z | Z | W | V | V |

Assuming that all candidates (or policies) that tie for the fewest votes are eliminated at the same time, under what conditions is an eventual majority winner guaranteed? Put another way, under what conditions might there not be an unambiguous majority winner? (Hint: How important is it for Evans, Foster, and Garcia to fill out their ballots completely?) How will these conditions change if Harris moves into town and votes?

## Problem 3: Auctions

In class professor mentioned that in a 2nd price "Vickrey" auction, it is best to tell the truth (as opposed to "shade"). Prove this is true. (Hint: show that bidding $b$ equal to the bidder's true value of the object $V$ weakly dominates any other strategy.)

## Problem 4: Uncertainty and Information

You are the Dean of the Faculty at St. Anford University. You hire Assistant Professors for a probationary period of 7 years, after which they come up for tenure and are either promoted and gain a job for life or turned down, in which case they must find another job elsewhere.

Your Assistant Professors come in two types, Good and Brilliant. Any types worse than Good have already been weeded out in the hiring process, but you cannot directly distinguish between Good and Brilliant types. Each individual Assistant Professor knows whether he or she is Brilliant or merely Good. You would like to tenure only the Brilliant types.

The payoff from a tenured career at St. Anford is $\$ 2$ million; think of this as the expected discounted present value of salaries, consulting fees, and book royalties, plus the monetary equivalent of the pride and joy that the faculty member and his or her family would get from being tenured at St. Anford. Anyone denied tenure at St. Anford will get a faculty position at Boondocks College, and the present value of that career is $\$ 0.5$ million.

Your faculty can do research and publish findings. But each publication requires effort and time and causes strain on the family; all these are costly to the faculty member. The monetary equivalent of this cost is $\$ 30,000$ per publication for a Brilliant Assistant Professor and $\$ 60,000$ per publication for a Good one. You can set a minimum number, N, of publications that an Assistant Professor must produce in order to achieve tenure.
(a) For what values of N will there be a pooling equilibrium where every Assistant Professor goes to Boondocks College?
(b) For what values of N will there be a pooling equilibrium where every Assistant Professor stays at St. Anford?
(b) What values of N will accomplish your goal of screening the Brilliant professors out from the merely Good ones?

